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# **Estimating the Heating Seasonal Operating Cost of Residential Hybrid Heat Pump Systems, Including Units Retrofitted to Oil, Gas and Electric Furnaces**

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National Bureau of Standards  
U.S. Department of Commerce  
Washington, DC 20234

July 1980

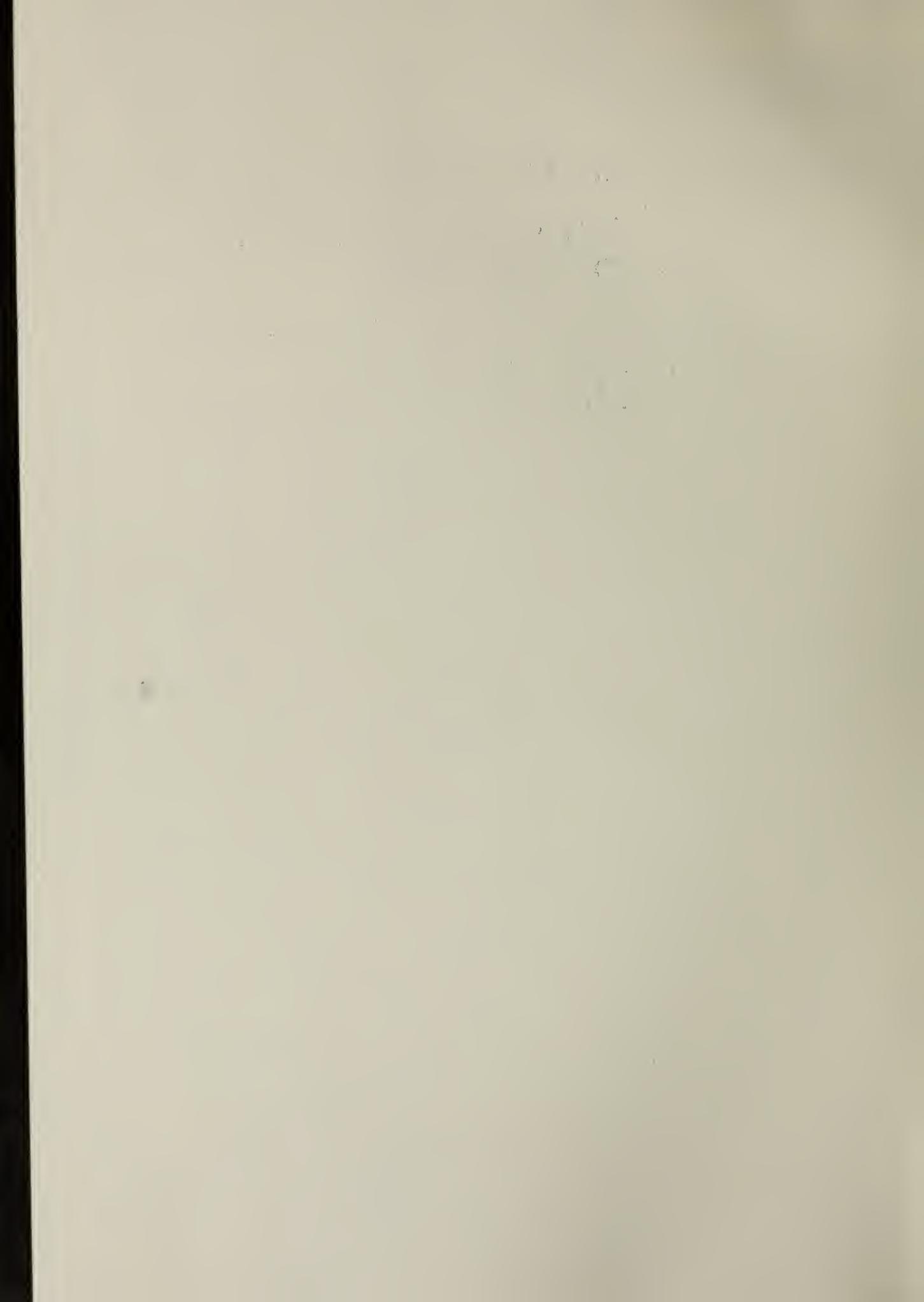


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OPERATING COST OF RESIDENTIAL  
HYBRID HEAT PUMP SYSTEMS,  
INCLUDING UNITS RETROFITTED TO OIL,  
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ESTIMATING THE HEATING SEASONAL OPERATING COST  
OF RESIDENTIAL HYBRID HEAT PUMP SYSTEMS,  
INCLUDING UNITS RETROFITTED TO OIL, GAS AND ELECTRIC FURNACES

by

Peter Domanski and George E. Kelly

ABSTRACT

A method is presented for estimating the heating seasonal operating cost of a residential, hybrid heating system consisting of an electric heat pump and a warm-air furnace. The approach described is applicable to heat pump/control system/gas or oil-fired furnace which is sold as a package or to a heat pump/control system which is intended to be added to an existing gas, oil or electric furnace. Recommendations are made regarding how such systems can be rated and the type of information that would assist consumers in comparing the operating cost of a hybrid heat pump system with that of a conventional heat pump or furnace. Different control strategies are accounted for and examples are presented (in the appendix) for estimating the heating seasonal operating cost of hybrid systems employing both single and two-speed compressors.

Key Words: Add-on heat pumps; furnaces; heat pumps; hybrid heat pumps;  
hybrid systems; rating procedure; seasonal cost of operation

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## 1. INTRODUCTION

The test and calculation procedures recommended herein apply to residential, hybrid heat pump systems. A hybrid heat pump system is defined in this report as either (1) a heat pump/control system/gas or oil-fired furnace which is sold as a package or (2) a heat pump/control system which is intended to be added to a previously installed gas, oil or electric furnace. This definition differs slightly from the usual definition of a hybrid system by the inclusion of the case where a heat pump is added to an existing electric furnace. This modified definition is adopted because it is felt that distinguishing between fossil-fuel and electric furnaces in the heat pump add-on application is largely academic, since both types of furnaces will tend to be operated in the same manner. In addition, since a heat pump retrofitted to an electric furnace will usually operate differently than a heat pump sold with electric resistance heaters, it is important to have a procedure for determining the heating seasonal operating cost of such add-on systems.

### 1.1 RECOMMENDED TEST PROCEDURE

A hybrid heat pump system which is intended to be sold with a particular gas or oil-fired furnace, shall be tested in accordance with: the Department of Energy's "Test Procedures for Central Air Conditioners, Including Heat Pumps" [1], and the Department of Energy's "Test Procedures for Furnaces and Vented Home Heating Equipment" [2]. The latter provides rating data for the furnace: efficiency number,  $EFFY_A$ , and an output capacity,  $Q_{out}$ , which are used in the calculation procedure described in Section 2 of this report to estimate the heating seasonal operating cost of the hybrid system.

A hybrid heat pump system which is intended to be sold as an add-on to a previously installed gas, oil, or electric furnace, need only be tested with the Department of Energy's "Test Procedures for Central Air Conditioners, Including Heat Pumps." However, in the case of these add-on units, the following furnace efficiencies,  $\eta_F$ , shall be used in Section 2 to calculate the heating seasonal operating cost of the hybrid system.

Type of Existing Furnace	Assigned Furnace Efficiency, $\eta_F$ (%)
gas-fueled furnace	62
oil-fueled furnace	68
electric furnace	100

In addition, an output capacity of the furnace,  $\dot{Q}_F$ , is assumed in Section 2 to be equal to 1.7 x DHR (the design heating requirement), which is equivalent to saying that the furnace is 70% oversized relative to the heating requirements of the residence at the outdoor design temperature.

## 1.2 DESCRIPTION OF RECOMMENDED RATING PROCEDURE

An analysis of the seasonal efficiency and seasonal operating cost for hybrid systems has shown that for a system employing a fossil-fuel furnace, the seasonal efficiency is not necessarily consistent with seasonal operating cost due to different control systems which may be employed and the wide range of prices for electricity and fuel. For example, a low-efficiency furnace may cost less to operate for the same heating done than a high-efficiency heat pump, if the price of electricity is high and the price of fuel low. This could result in a hybrid system with a higher efficiency number costing more to operate than a hybrid system with a low efficiency number, if the latter employs better system control

strategy. Since the consumer is primarily interested in what the cost of heating his house will be, a seasonal efficiency number may be misleading. For this reason, it is recommended that only Heating Seasonal Operating Cost (HSOC) be used to rate a hybrid heat pump system.

A procedure for calculating the HSOC for a hybrid unit is presented in Section 2. This calculation procedure recognizes that the hybrid heat pump system can operate in different modes, depending on the control system employed. Generally four modes of operation were found to be possible and the recommended calculation procedure can be applied to the particular type of control strategy used.\* These four modes of operation are:

- above balance point\*\* - (1) heat pump operates alone, the furnace is off
- below balance point - (2) heat pump is off, furnace operates alone
- (3) heat pump cycles with furnace to meet heating load (e.g., when furnace is on, heat pump is off and vice versa)
- (4) heat pump works continuously while furnace cycles to meet the heating load.

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\* Some simplifications are assumed in the recommended calculation procedure. First, the effect of a heat pump timer which results in a minimum off-period between compressor shut-off and start-up, is neglected. Second, the performance degradation of the heat pump when it cycles with the furnace is considered to equal the performance degradation of heat pump cycling by itself. However, results of several detailed HSOC calculations done without these two simplifications have shown that their effect on the final results is negligible.

\*\* The balance point is the outdoor temperature at which capacity of the heat pump equals the building heating load.

## 2. CALCULATION OF THE HEATING SEASONAL OPERATING COST

The heating seasonal operating cost, HSOC, of a hybrid system is strongly dependent upon the climatic region in which the unit operates, the type of system employed (e.g., single-speed or variable-speed compressor, type of furnace control strategy etc.), the design heating requirement\* of the building relative to the heat pump's capacity at different outdoor temperatures, and the cost of fuel and electricity. Because of these factors, it is recommended that the seasonal cost of operation of such a system be determined in each climatic region at a number of different design heating requirements and for a variety of different fuel and electric costs.

### 2.1 CALCULATING THE HEATING SEASONAL OPERATING COST FOR A HYBRID SYSTEM EMPLOYING A SINGLE SPEED COMPRESSOR

Table 1 lists six major U.S. climatic regions and their associated heating load hours, outdoor design temperatures, and fractional hours in each temperature bin. Figure 1 is a map of heating load hours (HLH) for the continental United States that may be used to locate these six regions. The minimum and maximum design heating requirements of a residence in which a hybrid system is likely to be installed will depend upon the climatic region and the capacity of the heat pump. They may be obtained for each of these six climatic regions by using the following equations:

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\* The "design heating requirement" is the heating requirement that must be met by the system at the 97-1/2 percent outdoor design temperature. A consumer purchasing a heat pump which provides both heating and cooling should be reminded to select a unit having a cooling capacity which will satisfy his home's cooling requirements and maintain a comfortable indoor level of relative humidity.

$$(\text{minimum design heating requirement}) = \begin{cases} \dot{Q}_{SS}(47) \frac{(65-T_{OD})}{60}, & \text{for regions I, II, III, IV, and VI} \\ \dot{Q}_{SS}(47) & \text{for region V} \end{cases} \quad (2.1)$$

and

$$(\text{maximum design heating requirement}) = \begin{cases} 2\dot{Q}_{SS}(47) \frac{(65-T_{OD})}{60}, & \text{for regions I, II, III, IV, and VI} \\ 2.2\dot{Q}_{SS}(47), & \text{for region V} \end{cases} \quad (2.2)$$

and rounding the results off to the nearest standardized design heating requirement given in Table 2. In the above equations,  $T_{OD}$  is the outdoor design temperature given in Table 1 for each major climatic region and  $\dot{Q}_{SS}(47)$  is the heat pump capacity measured during the high temperature performance test at 47°F.

Standard design heating requirements in a particular climatic region for which calculations should be performed are those given Table 2 ranging from the  $DHR_{min}$  to the  $DHR_{max}$  for the region.

For each climatic region and for each design heating requirement the heating seasonal operating cost, HSOC, of a hybrid system employing a single-speed compressor shall be determined using the following equation:

$$HSOC = \frac{HLH \times C \times DHR}{\sum_j \frac{n_j}{N} BL(T_j)} \left[ \sum_j \frac{n_j}{N} \frac{\delta(T_j)X(T_j)\gamma(T_j)\dot{E}(T_j)}{PLF(X,\gamma)} \times \frac{\$ \text{ cost}}{kWh} \right. \\ \left. + \sum_j \frac{n_j}{N} [BL(T_j) - \dot{Q}(T_j)\delta(T_j)X(T_j)\gamma(T_j)] \frac{100}{\eta_F} \frac{1000}{K} \times \$ \frac{\text{cost}}{\text{unit of fuel}} \right] \quad (2.3)$$

The first term on the right side of the above equation represents the operating cost of the heat pump, while the second term is the operating cost of the furnace. The ratio of  $(HLH \times C \times DHR)$  to  $(\sum_j \frac{n_j}{N} BL(T_j))$  is a

scaling factor which allows the operating cost of the hybrid system to be compared with that of a system employing either a single heat pump or a single furnace.

The symbols used in eq. (2.3) have the following meaning:

$j = 1, 2, 3, \dots, n$  corresponds to the  $j^{\text{th}}$  temperature bin

$n =$  total number of non-zero temperature bins in the climatic region

$T_j = 67 - 5j$  is the representative temperature of the  $j^{\text{th}}$  bin, ( $^{\circ}\text{F}$ )

$\Sigma_j$  indicates the quantity following the symbol is to be summed over all temperature bins

$\frac{n_j}{N} =$  the number of hours in the  $j^{\text{th}}$  temperature bin divided by  $N \equiv \Sigma_j n_j$ ; it is referred to as the "fractional hours in the  $j^{\text{th}}$  temperature bin" and values for it are given in Table 1 for each region

HLH = the number of heating load hours for the region as given in Table 1

DHR = the design heating requirement (kBtu/h)

$T_{\text{OD}}$  = the outdoor design temperature given in Table 1 for each major climatic region, ( $^{\circ}\text{F}$ )

$C = 0.77$ , an experience factor which tends to improve the agreement between calculated and measured building loads

$K =$  the Btu content per unit of fuel (e.g.  $K = 100,000$  Btu/therm if cost/unit of fuel is given in dollars per therm)

$\eta_{\text{F}}$  = the efficiency of the furnace, (%). For hybrid units sold with a particular furnace,  $\eta_{\text{F}}$  shall be set equal to  $\text{EFFY}_A$ , as determined by DOE "Test Procedure for Furnaces or Vented Home Heating Equipment" [2]. In a hybrid system intended to be added to an existing gas, oil or electric furnace, the values assigned in Section 1.1 for  $\eta_{\text{F}}$  shall be employed.

$\dot{Q}(T_j)$  = the heat pump's capacity at temp.  $T_j$  (kBtu/h)

$\dot{E}(T_j)$  = the heat pump's power at temp.  $T_j$  (kW)

$BL(T_j)$  = the building load at temp.  $T_j$  (kBtu/h) as defined in eq. (2.6)

$\delta(T_j)$  = the heat pump's cut-out factor due to low temperature or heat pump/furnace control strategy as defined in eq. (2.7)

$X(T_j)$  = the heat pump's heating-load factor at temp.  $T_j$  as defined in eq. (2.8)

$\gamma(T_j)$  = the furnace-heat pump cycling factor at temp.  $T_j$  as defined in eq. (2.9)

$PLF(X, \gamma)$  = the heat pump's part-load factor as defined in eq. (2.11).

The heat pump's capacity and power at temperature  $T_j$  shall be estimated using the following formulas:

$$\dot{Q}(T_j) = \begin{cases} \dot{Q}_{SS}(17) + \frac{(\dot{Q}_{SS}(47) - \dot{Q}_{SS}(17)) (T_j - 17)}{30}, & T_j \geq 45^\circ\text{F or } T_j \leq 17^\circ\text{F} \\ \dot{Q}_{SS}(17) + \frac{(\dot{Q}_{DEF}(35) - \dot{Q}_{SS}(17)) (T_j - 17)}{18}, & 17^\circ\text{F} < T_j < 45^\circ\text{F} \end{cases} \quad (2.4)$$

$$\dot{E}(T_j) = \begin{cases} \dot{E}_{SS}(17) + \frac{(\dot{E}_{SS}(47) - \dot{E}_{SS}(17)) (T_j - 17)}{30}, & T_j \geq 45^\circ\text{F or } T_j \leq 17^\circ\text{F} \\ \dot{E}_{SS}(17) + \frac{(\dot{E}_{DEF}(35) - \dot{E}_{SS}(17)) (T_j - 17)}{18}, & 17^\circ\text{F} < T_j < 45^\circ\text{F} \end{cases} \quad (2.5)$$

where  $\dot{Q}_{SS}(47)$  and  $\dot{E}_{SS}(47)$ ,  $\dot{Q}_{DEF}(35)$  and  $\dot{E}_{DEF}(35)$ , and  $\dot{Q}_{SS}(17)$  and  $\dot{E}_{SS}(17)$  are the capacities (in kBtu/h) and powers (in kW) measured during the high temperature test, the frost accumulation test, and the low temperature test, respectively.\* The quantities  $BL(T_j)$ ,  $\delta(T_j)$ ,  $\gamma(T_j)$  and  $PLF(X, \gamma)$  are

\* Refer to DoE's "Test Procedures for Central Air Conditioners, Including Heat Pumps." [1]

defined by the following equations:

$$BL(T_j) = \frac{65 - T_j}{65 - T_{OD}} \times C \times DHR \quad (2.6)$$

$$\delta(T_j) = \begin{cases} 0 & ; & T_j \leq T_{OFF} & \text{or} & \frac{\dot{Q}(T_j)}{(3.413)(\dot{E}(T_j))} < 1 \\ \frac{1}{2} & ; & T_{OFF} < T_j \leq T_{ON} & \text{and} & \frac{\dot{Q}(T_j)}{(3.413)(\dot{E}(T_j))} \geq 1 \\ 1 & ; & T_j > T_{ON} & \text{and} & \frac{\dot{Q}(T_j)}{(3.413)(\dot{E}(T_j))} > 1 \end{cases} \quad (2.7)$$

where  $T_{OFF}$  and  $T_{ON}$  denote:

$T_{OFF}$  = the outdoor temperature at which the compressor is automatically shut off (if no such temperature exists,  $T_j$  is always greater than  $T_{OFF}$  and  $T_{ON}$ ) due to low temperature or heat pump/furnace control strategy

$T_{ON}$  = the outdoor temperature at which the compressor is automatically turned on (if applicable) for units designed for low temperature automatic shut-off or due to heat pump/furnace control strategy.

$$X(T_j) = \begin{cases} \frac{BL(T_j)}{\dot{Q}(T_j)} & ; & \dot{Q}(T_j) > BL(T_j) \\ 1 & ; & \dot{Q}(T_j) \leq BL(T_j) \end{cases} \quad (2.8)$$

$$\gamma(T_j) = \begin{cases} \frac{\dot{Q}_F - BL(T_j)}{Q_F - Q(T_j)} & , & \text{when } BL(T_j) > \dot{Q}(T_j) \text{ and cycling} \\ & & \text{between the furnace \& heat pump} \\ & & \text{occurs at temperature } T_j \\ 1 & , & \text{otherwise} \end{cases} \quad (2.9)$$

$$\text{where } \dot{Q}_F = \begin{cases} Q_{out}, \text{ which is defined in sections 4.7 and 4.10 of} \\ \text{Appendix N. Federal Register Vol. 43, No. 91,} \\ \text{May 10, 1978 (p. 20164)} \\ (1.7) \times (DHR) \text{ for a heat pump intended to be sold as} \\ \text{an add-on to previously installed furnace} \end{cases} \quad (2.10)$$

$$PLF(X, \gamma) = 1 - C_D(1 - (X)(\gamma)) \quad (2.11)$$

where  $C_D$  is degradation coefficient which is either measured or assumed in accordance with DOE's "Test Procedure for Central Air Conditioners, Including Heat Pumps" [1].

The current (1980) recommended fuel and electricity price ranges for which the above calculations are to be performed are:

electricity (\$/kWh)	.04,	.06,	.08,	.10,		
oil (\$/gallon)	.40,	.60,	.80,	1.00,	1.20	1.40

The above prices for oil give the same HSOC for the following prices (in \$/therm) of gas, respectively .26, .39, .52, .65, .78, .91 taking into account the furnace efficiencies of  $\eta_{Foil} = 0.68$  and  $\eta_{FGas} = 0.62$ , as assigned in Section 1.1.

It is recommended that all HSOC figures be rounded off to the nearest five dollars and the information for each region be arranged in form similar to Tables 3 and 4. Appendix A presents step-by-step sample HSOC calculations according to the method described above.

## 2.2 Calculating the Heating Seasonal Operating Cost for Hybrid Systems with Two-Speed Compressor or Two Compressors

The minimum and maximum design heating requirements of a residence in which a heat pump is likely to be installed, shall be determined for the six climatic regions listed in Table 1 using the same procedure outlined in Section 2.1 for units with single speed compressors. The only difference is that  $\dot{Q}_{SS}^{(k=2)}$  (47) (which is the capacity measured in the high temperature performance test at 47°F (+8.3 °C) with the unit operating at the high compressor

speed or with both compressors in operation) shall be used in place of  $\dot{Q}_{SS}(47)$  in the equations (2.1) and (2.2) for the maximum and minimum design heating requirements.

For each climatic region and for each standard DHR ranging from the minimum to the maximum design heating requirement, the heating seasonal operating cost, HSOC, shall be determined using the following equation:

$$HSOC = \frac{HLH \times C \times DHR}{\sum_j \frac{n_j}{N} BL(T_j)} \left[ \sum_j \frac{E_{HP}(T_j)}{N} \times \frac{\$ \text{ cost}}{kW} + \sum_j \frac{E_F(T_j)}{N} \frac{100}{\eta_F} \frac{1000}{K} \times \frac{\$ \text{ cost}}{\text{unit of fuel}} \right] \quad (2.12)$$

where  $\frac{E_{HP}(T_j)}{N}$  = energy used by a heat pump in the  $j^{\text{th}}$  temperature bin divided by the total number of bin hours, and

$\frac{E_F(T_j)}{N}$  = the heat supplied by a furnace in the  $j^{\text{th}}$  temperature bin divided by the total number of bin hours.

The terms  $PLF^{k=1}$ ,  $PLF^{k=2}$ ,  $X^{k=1}$ ,  $X^{k=2}$ , and  $\gamma^{k=2}$ , which are used below, are consistent with the definitions employed for single-speed compressor systems, but with their meaning expanded to including high speed or two compressor operation ( $k=2$ ) or low speed or single compressor operation ( $k=1$ ). These terms, along with the terms  $\frac{E_{HP}(T_j)}{N}$  and  $\frac{E_F(T_j)}{N}$ , are evaluated according to the four possible cases of heat pump operation denoted below.

#### CASE 1

$BL(T_j) \leq \dot{Q}^{k=1}(T_j)$  and the heat pump operates at low compressor speed or with a single compressor.

$$\frac{E_{HP}(T_j)}{N} = \frac{n_j}{N} \frac{X^{k=1}(T_j)}{PLF^{k=1}} \delta'(T_j) E^{k=1}(T_j) \quad (2.13)$$

$$\frac{E_F(T_j)}{N} = \frac{n_j}{N} BL(T_j) [1 - \delta'(T_j)] \quad (2.14)$$

$$\text{where } X^{k=1}(T_j) = \frac{BL(T_j)}{\dot{Q}^{k=1}(T_j)} \quad (2.15)$$

$$PLF^{k=1} = 1 - C_D^{k=1} [1 - X^{k=1}(T_j)] \quad (2.16)$$

$$\delta'(T_j) = \begin{cases} 0 & ; \quad T_j \leq T_{OFF} \\ \frac{1}{2} & ; \quad T_{OFF} < T_j \leq T_{ON} \\ 1 & ; \quad T_j > T_{ON} \end{cases} \quad (2.17)$$

## CASE 2

$\dot{Q}^{k=1}(T_j) < BL(T_j) < \dot{Q}^{k=2}(T_j)$  and the heat pump alternates between high speed or two-compressor operation ( $k=2$ ) and low speed or single-compressor operation ( $k=1$ )

$$\frac{E_{HP}(T_j)}{N} = \frac{n_j}{N} \delta'(T_j) [\dot{E}^{k=1}(T_j) X^{k=1}(T_j) + \dot{E}^{k=2}(T_j) X^{k=2}(T_j)] \quad (2.18)$$

$$\frac{E_F(T_j)}{N} = \frac{n_j}{N} BL(T_j) [1 - \delta'(T_j)] \quad (2.19)$$

$$\text{where } X^{k=1}(T_j) = \frac{\dot{Q}^{k=2}(T_j) - BL(T_j)}{\dot{Q}^{k=2}(T_j) - \dot{Q}^{k=1}(T_j)} \quad (2.20)$$

$$X^{k=2}(T_j) = 1 - X^{k=1}(T_j) \quad (2.21)$$

$$\delta^{\cdot}(T_j) = \begin{cases} 0 & ; & T_j \leq T_{\text{OFF}} \\ \frac{1}{2} & ; & T_{\text{OFF}} < T_j \leq T_{\text{ON}} \\ 1 & ; & T_j > T_{\text{ON}} \end{cases} \quad (2.22)$$

### CASE 3

$\dot{Q}^{k=1}(T_j) < BL(T_j) < \dot{Q}^{k=2}(T_j)$  and the heat pump cycles on and off at high compressor speed, or cycles both compressors on and off together ( $k=2$ )

$$\frac{E_{\text{HP}}(T_j)}{N} = \frac{n_j}{N} \frac{X^{k=2}(T_j)}{PLF^{k=2}(T_j)} \delta^{\cdot}(T_j) \dot{E}^{k=2}(T_j) \quad (2.23)$$

$$\frac{E_{\text{F}}(T_j)}{N} = \frac{n_j}{N} BL(T_j) [1 - \delta^{\cdot}(T_j)] \quad (2.24)$$

$$\text{where } X^{k=2}(T_j) = \frac{BL(T_j)}{\dot{Q}^{k=2}(T_j)} \quad (2.25)$$

$$PLF^{k=2}(T_j) = 1 - C_D^{k=2} (1 - X^{k=2}(T_j)) \quad (2.26)$$

$$\delta^{\cdot}(T_j) = \begin{cases} 0 & ; & T_j \leq T_{\text{OFF}} \\ \frac{1}{2} & ; & T_{\text{OFF}} < T_j \leq T_{\text{ON}} \\ 1 & ; & T_j > T_{\text{ON}} \end{cases} \quad (2.27)$$

### CASE 4

$BL(T_j) > \dot{Q}^{k=2}(T_j)$  and the heat pump alternates with the furnace, or the heat pump works continuously while furnace cycles to meet building heating load, or the furnace operates alone.

$$\frac{E_{HP}(T_j)}{N} = \frac{n_j}{N} \frac{\gamma^{k=2}(T_j)}{PLF^{k=2}(T_j)} \delta''(T_j) \dot{E}^{k=2}(T_j) \quad (2.28)$$

$$\frac{E_F(T_j)}{N} = \frac{n_j}{N} [BL(T_j) - \dot{Q}^{k=2}(T_j) \delta''(T_j) \gamma^{k=2}(T_j)] \quad (2.29)$$

where

$$\gamma^{k=2}(T_j) = \begin{cases} \frac{\dot{Q}_F - BL(T_j)}{\dot{Q}_F - \dot{Q}^{k=2}(T_j)} & \text{when cycling occurs} \\ & \text{between the heat pump} \\ & \text{and the furnace} \\ 1 & \text{otherwise} \end{cases} \quad (2.30)$$

$$PLF^{k=2}(T_j) = 1 - C_D (1 - \gamma^{k=2}(T_j)) \quad (2.31)$$

$$\delta''(T_j) = \begin{cases} 0 & ; \quad T_j \leq T_{OFF} \text{ or } \frac{\dot{Q}^{k=2}(T_j)}{(3.413)(\dot{E}^{k=2}(T_j))} < 1 \\ \frac{1}{2} & ; \quad T_{OFF} < T_j \leq T_{ON} \text{ and } \frac{\dot{Q}^{k=2}(T_j)}{(3.413)(\dot{E}^{k=2}(T_j))} > 1 \\ 1 & ; \quad T_j > T_{ON} \text{ and } \frac{\dot{Q}^{k=2}(T_j)}{(3.412)(\dot{E}^{k=2}(T_j))} > 1 \end{cases} \quad (2.32)$$

In each of the above cases, the heating capacity  $\dot{Q}(T_j)$ , in kBtu/h, and the power input  $\dot{E}^k(T_j)$ , in kW, corresponding to low speed or single-compressor operation ( $k=1$ ), and high speed or two-compressor operation ( $k=2$ ), shall be calculated (when required) as follows:

$$\dot{Q}^{k=1}(T_j) = \begin{cases} \dot{Q}_{SS}^{k=1}(47) + \frac{[\dot{Q}_{SS}^{k=1}(62) - \dot{Q}_{SS}^{k=1}(47)](T_j - 47)}{15}; & T_j \geq 40^\circ\text{F} \\ \dot{Q}_{SS}^{k=1}(17) + \frac{[\dot{Q}_{DEF}^{k=1}(35) - \dot{Q}_{SS}^{k=1}(17)](T_j - 17)}{18}; & 17^\circ\text{F} \leq T_j < 40^\circ\text{F} \\ \dot{Q}_{SS}^{k=1}(17) + \frac{[\dot{Q}_{SS}^{k=1}(47) - \dot{Q}_{SS}^{k=1}(17)](T_j - 17)}{30}; & T_j < 17^\circ\text{F} \end{cases} \quad (2.33)$$

$$\dot{Q}^{k=2}(T_j) = \begin{cases} \dot{Q}_{SS}^{k=2}(17) + \frac{[\dot{Q}_{SS}^{k=2}(47) - \dot{Q}_{SS}^{k=2}(17)](T_j - 17)}{30}; & T_j \geq 45^\circ\text{F} \text{ or } T_j \leq 17^\circ\text{F} \\ \dot{Q}_{SS}^{k=2}(17) + \frac{[\dot{Q}_{DEF}^{k=2}(35) - \dot{Q}_{SS}^{k=2}(17)](T_j - 17)}{18}; & 17^\circ\text{F} < T_j < 45^\circ\text{F} \end{cases} \quad (2.34)$$

$$\dot{E}^{k=1}(T_j) = \begin{cases} \dot{E}_{SS}^{k=1}(47) + \frac{[\dot{E}_{SS}^{k=1}(62) - \dot{E}_{SS}^{k=1}(47)](T_j - 47)}{15}; & T_j \geq 40^\circ\text{F} \\ \dot{E}_{SS}^{k=1}(17) + \frac{[\dot{E}_{DEF}^{k=1}(35) - \dot{E}_{SS}^{k=1}(17)](T_j - 17)}{18}; & 17^\circ\text{F} \leq T_j < 40^\circ\text{F} \\ \dot{E}_{SS}^{k=1}(17) + \frac{[\dot{E}_{SS}^{k=1}(47) - \dot{E}_{SS}^{k=1}(17)](T_j - 17)}{30}; & T_j < 17^\circ\text{F} \end{cases} \quad (2.35)$$

$$\dot{E}^{k=2}(T_j) = \begin{cases} \dot{E}_{SS}^{k=2}(17) + \frac{[\dot{E}_{SS}^{k=2}(47) - \dot{E}_{SS}^{k=2}(17)](T_j - 17)}{30}; & T_j \geq 45^\circ\text{F} \text{ or } T_j \leq 17^\circ\text{F} \\ \dot{E}_{SS}^{k=2}(17) + \frac{[\dot{E}_{DEF}^{k=2}(35) - \dot{E}_{SS}^{k=2}(17)](T_j - 17)}{18}; & 17^\circ\text{F} < T_j < 45^\circ\text{F} \end{cases} \quad (2.36)$$

where  $\dot{Q}_{SS}^k(62)$  and  $\dot{E}_{SS}^k(62)$ ,  $\dot{Q}_{SS}^k(47)$  and  $\dot{E}_{SS}^k(47)$ ,  $\dot{Q}_{DEF}^k(35)$  and  $\dot{E}_{DEF}^k(35)$ , and  $\dot{Q}_{SS}^k(17)$  and  $\dot{E}_{SS}^k(17)$  are the capacities (in kBtu/h) and power (in kW) measured during the high temperature tests at 62°F, the high temperature tests at 47°F, the frost accumulation tests at 35°F, and the low temperature tests at 17°F, respectively.\* It should be noted that if these definitions of  $\dot{Q}^k(T_j)$  and  $\dot{E}^k(T_j)$  result in the quantity  $\frac{\dot{Q}^{k=2}(T_j)}{(3.413)(\dot{E}^{k=2}(T_j))}$  being less than unity for

a temperature  $T_j$ , then the value of  $\delta''(T_j)$  used in the above HSOC calculation are set equal to zero at this particular temperature  $T_j$ . This avoids the possibility of having an efficiency for the heat pump at an outdoor temperature  $T_j$  which is less than unity because of any errors introduced by the straight line extrapolation of the measured capacities and power inputs to low outdoor temperatures.

In the four cases described above,  $T_{OFF}$  and  $T_{ON}$  are, respectively, the outdoor temperatures at which compressor operation automatically stops and automatically starts. If no such temperatures exists, then  $T_j$  is always greater than  $T_{ON}$  and  $T_{OFF}$ . The quantity  $C_D^{k=2}$  is the part load degradation factor for the unit cycling at high compressor speed or with both compressors simultaneously cycling (if applicable), and  $C_D^{k=1}$  is the part load degradation factor for unit cycling at low compressor speed or with the single compressor that normally operates at low heating loads (high outdoor temperatures).

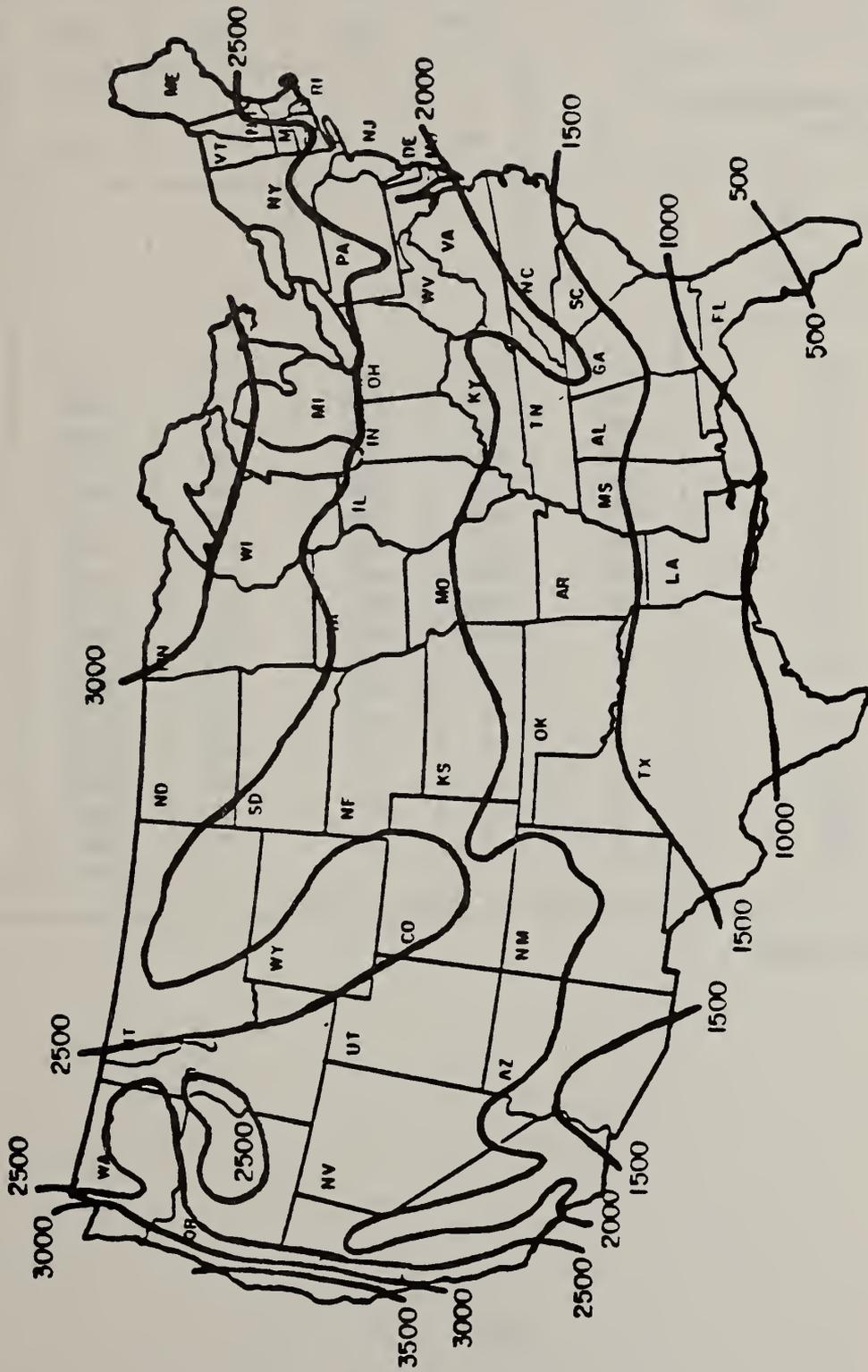
Appendix B presents a sample step-by-step calculation of the HSOC according to the procedure described above.

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\* Refer to DoE's "Test Procedure for Central Air Conditioners, Including Heat Pumps." [1]

## REFERENCES

1. "Test Procedures for Central Air Conditioners, Including Heat Pumps", Federal Register, Vol. 44, No. 249, Dec. 27, 1979, pages 76700 through 76716.
2. "Test Procedures for Furnaces and Vented Home Heating Equipment", Federal Register, Vol. 42, No. 91, May 10, 1978, pages 20147 through 20181.



This map is reasonably accurate for most parts of the United States but is necessarily highly generalized and consequently not too accurate in mountainous regions particularly in the Rockies

Figure 1. Heating Load Hours (HLH) for the United States. [1]

Table 1. Major Climatic Regions In the Continental U.S.A. [1]

Region		I	II	III	IV	V	VI
Heating Load Hours, HLH		750	1250	1750	2250	2750	2750*
Outdoor Design Temperature, $T_{OD}$ for the region		37	27	17	5	-10	30
Fractional Hours, $\frac{n_j}{N}$ :							
Bin #	$T_j$ ( $^{\circ}$ F)						
j = 1	62	.291	.215	.153	.132	.106	.113
2	57	.239	.189	.142	.111	.092	.206
3	52	.194	.163	.138	.103	.086	.215
4	47	.129	.143	.137	.093	.076	.204
5	42	.081	.112	.135	.100	.078	.141
6	37	.041	.088	.118	.109	.087	.076
7	32	.019	.056	.092	.126	.102	.034
8	27	.005	.024	.047	.087	.094	.008
9	22	.001	.008	.021	.055	.074	.003
10	17	0	.002	.009	.036	.055	0
11	12	0	0	.005	.026	.047	0
12	7	0	0	.002	.013	.038	0
13	2	0	0	.001	.006	.029	0
14	-3	0	0	0	.002	.018	0
15	-8	0	0	0	.001	.010	0
16	-13	0	0	0	0	.005	0
17	-18	0	0	0	0	.002	0
18	-23	0	0	0	0	.001	0

\* In Pacific Coast region

**Table 2. Standardized Design Heating Requirements (kBtu/h)**

5	25	50	90
10	30	60	100
15	35	70	110
20	40	80	130

Table 3. Example of Information Which Would Assist a Consumer in Purchasing a Heat Pump/Oil Furnace System.

Cost of oil \$/Gallon Cost of electricity \$/kWh DHR kBtu/h		Heating Seasonal Operating Cost (\$)				
		.80	1.00	1.20	1.40	
REGION I	10	.04 .06 .08 .10				
	15	.04 .06 .08 .10				
	20	.04 .06 .08 .10				
	25	.04 .06 .08 .10				
	30	.04 .06 .08 .10				

Table 4. Example of Information Which Would Assist a Consumer in Purchasing an Add-on Heat Pump.

Cost of gas \$/therm Cost of oil \$/gallon Cost of electricity \$/kWh DHR kBtu/h		Heating Seasonal Operating Cost (\$)						Add-on heat pump + electric furnace
		Add-on heat pump + gas or oil furnace						
		.26	.39	.52	.65	.78	.91	
REGION I	10	.04 .06 .08 .10						
	15	.04 .06 .08 .10						
	20	.04 .06 .08 .10						
	25	.04 .06 .08 .10						
	30	.04 .06 .08 .10						

## Appendix A

Sample of Heating Seasonal Operating Cost Calculations for a Hybrid System with a Single-Speed Compressor.

Sample calculations of Heating Seasonal Operating Cost for region V are given below to show the sequence of calculating steps. The data employed is for a fictitious hybrid system.

### Rating Data for heat pump

$$\dot{Q}_{SS}(47^{\circ}\text{F}) = 31.00 \text{ kBtu/h} \quad \dot{E}_{SS}(47^{\circ}\text{F}) = 3.430 \text{ kW}$$

$$\dot{Q}_{DEF}(35^{\circ}\text{F}) = 24.00 \text{ kBtu/h} \quad \dot{E}_{SS}(35^{\circ}\text{F}) = 3.150 \text{ kW}$$

$$\dot{Q}_{SS}(17^{\circ}\text{F}) = 17.00 \text{ kBtu/h} \quad \dot{E}_{SS}(17^{\circ}\text{F}) = 2.770 \text{ kW}$$

$$C_D = .25$$

### Rating data for furnace

Two examples are considered. The first assumes that a heat pump is added to a previously installed gas, oil or electric furnace with  $\eta_F$  equal to 62%, 68% or 100% respectively and capacity  $\dot{Q}_F = 1.7 \times \text{DHR} = 1.7 \times 70 = 119,000$  kBtu/h. The second example assumes that the hybrid system is sold with a particular oil-fired furnace model of capacity  $Q_{\text{out}} = 119$  kBtu/h and an  $\text{EFFY}_A = 70\%$  as determined from the DOE furnace test procedure.

### Control information

The heat pump is the only source of heat as long as it can handle the load alone. It thus supplies all the required heat above the balance point. Below the balance point, it alternates operation with the furnace down to the outdoor temperature  $T = 10^{\circ}\text{F}$ , below which the heat pump is

turned off and the furnace satisfies the whole heating load. The heat pump is turned on again at an outdoor temperature of  $T = 20^{\circ}\text{F}$ .

Thus  $T_{\text{OFF}} = 10^{\circ}\text{F}$  and  $T_{\text{ON}} = 20^{\circ}\text{F}$ .

Standard design heating requirements.

Using Equations (2.1), and (2.2) and Table 2, it is found for region V that:

$$\text{DHR}_{\text{min}} = \dot{Q}_{\text{SS}}(47^{\circ}\text{F}) = 31.00 \approx 30.00 \text{ kBtu/h}$$

$$\text{DHR}_{\text{max}} = 2.2 \dot{Q}_{\text{SS}}(47^{\circ}\text{F}) = 68.20 \approx 70.00 \text{ kBtu/h}$$

The standard DHR's for which calculations shall be performed will thus be 30, 35, 40, 50, 60, 70 Btu/h as per Table 2.

Calculating Seasonal Operating Cost for DHR = 70 kBtu/h.

Table A1 presents sample worksheet for evaluation of the energy input to the hybrid system for both considered cases for DHR = 70 kBtu/h in region V.

The number of heating load hours (HLH) and the design outdoor temperature ( $T_{\text{OD}}$ ) were read from Table 1. An example showing how the figures in the various columns were obtained is given below for the temperature bin  $T_j = 27^{\circ}\text{F}$ .

Column a, b and c : These values are read directly from Table 1

Column d : furnace capacity  $\dot{Q}_{\text{F}}$ , use eq. (2.10)

For example with a previously installed furnace

$$\dot{Q}_{\text{F}} = 1.7 \times 709 = 119 \text{ kBtu/h}$$

For example for the hybrid system sold with the

$$\text{particular furnace } \dot{Q}_{\text{F}} = Q_{\text{out}} = 119 \text{ kBtu/h}$$

so  $\dot{Q}_F = 119 \text{ kBtu/h}$  for both cases.

(Since the furnace capacities are equal, all calculations will be the same for both given examples for  $\text{DHR} = 70 \text{ kBtu/h}$ . In the calculations of the other DHR's the capacity assumed for the previously installed furnace will be changed ( $\dot{Q}_F = 1.7 \times \text{DHR}$ ), while the capacity of the second furnace will, of course, stay  $Q_{\text{out}} = 119 \text{ Btu/h}$ .)

Column e

: heat pump capacity  $\dot{Q}(T_j)$ , use eq. (2.4)

$$\begin{aligned}\dot{Q}(27) &= \dot{Q}_{\text{SS}}(17) + \frac{[\dot{Q}_{\text{DEF}}(35) - \dot{Q}_{\text{SS}}(17)] (27 - 17)}{18} = \\ &= 17.00 + \frac{(24.00 - 17.00) (27 - 17)}{18} = 20.88 \text{ kBtu/h}\end{aligned}$$

column f

: heat pump power  $\dot{E}(T_j)$ , use Eq. (2.5)

$$\begin{aligned}\dot{E}(27) &= \dot{E}_{\text{SS}}(17) + \frac{[\dot{E}_{\text{DEF}}(35) - \dot{E}_{\text{SS}}(17)] (27 - 17)}{18} = \\ &= 2.770 + \frac{(3.150 - 2.770) (27 - 17)}{18} = 2.981 \text{ kW}\end{aligned}$$

column g

: building load  $\text{BL}(T_j)$ , use eq. (2.6)

$$\begin{aligned}\text{BL}(27) &= \frac{65 - 27}{65 - (-10)} \times C \times \text{DHR} = \\ &= \frac{65 - 27}{65 - (-10)} \times .77 \times 70 = 27.31 \text{ kBtu/h}\end{aligned}$$

column h : heating load  $\frac{n_j}{N} BL(T_j)$

$$\frac{n_j}{N} BL(27) = .094 \times 27.31 = 2.5671 \text{ kBtu/h}$$

column i : low temperature cut-off factor, use eq. (2.7)

$$\delta(27) = 1$$

column j : heating load factor  $X(T_j)$ , use eq. (2.8)

$$X(27) = 1$$

column k : furnace - heat pump cycling factor  $\gamma(T_j)$ , use eq. (2.9)

$$\begin{aligned} \gamma(27) &= \frac{\dot{Q}_F - BL(27)}{\dot{Q}_F - \dot{Q}(27)} = \\ &= \frac{119.00 - 27.31}{119.00 - 20.89} = .93 \end{aligned}$$

column l : part load factor  $PLF(X, \gamma)$ , use eq. (2.11)

$$\begin{aligned} PLF(X, \gamma) &= 1 - C_D (1 - X(27) \gamma(27)) = \\ &= 1 - .25 (1 - 1 \times .93) = .98 \end{aligned}$$

column m : heat pump energy input

$$\begin{aligned} \frac{n_j}{N} \frac{\delta(27) X(27) \gamma(27)}{PLF(X, \gamma)} \dot{E}(27) &= \\ = .094 \frac{(1)(1)(.93)}{.98} 2.981 &= .2662 \text{ kW} \end{aligned}$$

column n : heating load handled by the furnace

$$\frac{n_j}{N} [BL(27) - \dot{Q}(27)\delta(27)X(27)\gamma(27)] =$$

$$= .094 \times [27.31 - 20.88 \times 1 \times 1 \times .93] = .7319 \frac{\text{kBtu}}{\text{h}}$$

The above calculations must be repeated for each temperature bin. Then, to obtain the heating seasonal operating cost, the sum of the results of column h, m, n in Table A1 have to be applied as indicated by equation (2.3) along with respective combinations of electricity and fuel prices. An example is given below for electricity cost \$.04/kWh, cost of oil \$1.00/gallon and cost of gas \$.26/therm.

Hybrid system sold with an oil furnace with  $\eta_F = 70\%$

$$\text{HSOC} = \frac{2750 \times .77 \times 70}{21.3084} \left[ 1.6314 \times .04 + 9.6934 \frac{100}{70} \times \frac{1000}{140,000} \times 1.00 \right] = \$1142$$

Hybrid system added to an existing gas-fired furnace

$$\text{HSOC} = \frac{2750 \times .77 \times 70}{21.3084} \left[ 1.6314 \times .04 + 9.6934 \frac{100}{62} \times \frac{1000}{100,000} \times .26 \right] = \$737$$

Hybrid system added to an existing electric furnace. In the case of heat pump being added to an existing system, the Heating Seasonal Operating Cost becomes:

$$\text{HSOC} = \frac{2750 \times .77 \times 70}{21.3084} \left[ 1.6314 \times .04 + 9.6934 \times \frac{1000}{3413} \times .04 \right] = \$1220$$

The above calculation procedure was also performed for other standard design heating requirements applicable to the analyzed system in region V eq. 30, 35, 40, 50 and 60 kBtu/h. The final results for region V, rounded

off to the nearest \$5 are shown in Tables A2 and A3. Table A2 presents results for a heat pump sold with a particular furnace of efficiency  $\eta_F = 0.70$ . Table A3 contains results for the same heat pump sold as an add-on to a previously installed oil, gas or electric furnace. Furnace efficiencies employed in the latter case are respectively  $\eta_F = 0.68$ , 0.62 and 1.

Table A1

Sample Worksheet Used to Evaluate the Heating Seasonal Energy Input to a Hybrid System with a Single-Speed-Compressor Heat Pump

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	
Bin number	Representative temperature $T_j$	Fractional $\frac{n_j}{N}$ bin hours	Furnace capacity $Q_F$ (kBtu/h)	Heat pump capacity $\dot{Q}(T_j)$ (kBtu/h)	Heat pump power (kW)	Building Load $BL(T_j)$ (kBtu/h)	Heating Load (kBtu/h)	$\delta(T_j)$	$X(T_j)$	$\gamma(T_j)$	PLF(X, $\gamma$ )	Heat pump energy input (kW)	Heating Load handled by furnace (kBtu/h)	
1	62	.106		38.00	3.760	2.16	.2285	1	.06	1	.76	.0295	-	
2	57	.092		35.67	3.650	5.75	.5289	1	.16	1	.79	.0684	-	
3	52	.086		33.33	3.540	9.34	.8034	1	.28	1	.82	.1040	-	
4	47	.076		31.00	3.430	12.94	.9831	1	.42	1	.85	.1273	-	
5	42	.078		26.72	3.298	16.53	1.2892	1	.62	1	.91	.1759	-	
6	37	.087		24.78	3.192	20.12	1.7506	1	.81	1	.95	.2366	-	
7	32	.102		22.83	3.087	23.72	2.4190	1	1	.99	1	.3127	.1118	
8	27	.094		20.89	2.981	27.31	2.5670	1	1	.93	.98	.2662	.7319	
9	22	.074	↑	18.94	2.876	30.90	2.2867	1	1	.88	.97	.1931	1.0528	
10	17	.055		17.00	2.77	34.50	1.8972	.5	1	.83	.96	.0659	1.5099	
11	12	.047		14.67	2.66	38.09	1.7901	.5	1	.77	.94	.0513	1.5228	
12	7	.038	119		2.55	41.68	1.5839	0	1	1	1	.0	1.5839	
13	2	.029				45.27	1.3130	0	1	1	1	.0	1.3130	
14	-3	.018	↓			48.86	0.8796	0	1	1	1	.0	0.8796	
15	-8	.010				52.46	0.5246	0	1	1	1	.0	0.5246	
16	-13	.005				56.06	0.2803	0	1	1	1	.0	0.2803	
17	-18	.002				59.65	0.1193	0	1	1	1	.0	0.1193	
18	-23	.001				63.24	0.0632	0	1	1	1	.0	0.0632	
							21.3084						1.6314	9.6934

Region V

HLH = 2750

$$\sum_j \frac{n_j}{N} BL(T_j) = 21.3084 \text{ kBtu/h}$$

T<sub>OD</sub> = -10°F

$$\sum_j \frac{n_j}{N} \frac{\delta(T_j) X(T_j) \gamma(T_j)}{PLF(X, \gamma)} \dot{E}(T_j) = 1.6314 \text{ kW}$$

DHR = 70 kBtu/h

$$\sum_j \frac{n_j}{N} [BL(T_j) - \dot{Q}(T_j) \delta(T_j) X(T_j) \gamma(T_j)] = 9.6934 \text{ kBtu/h}$$

Table A2

Heating Seasonal Operating Cost\* of Hybrid System with an Oil-fired Furnace\* in Region V.

		Heat pump + oil furnace				
		.80	1.00	1.20	1.40	
Region V	DHR kBtu/h	Cost of oil \$/gallon				
		Cost of electricity \$/kWh				
Region V	30	.04	435	475	515	560
		.06	570	610	655	695
		.08	705	750	790	830
		.10	845	885	925	965
	35	.04	500	550	600	650
		.06	655	705	755	800
		.08	810	860	905	955
		.10	965	1010	1060	1090
	40	.04	570	630	685	740
		.06	740	795	855	910
		.08	910	965	1025	1080
		.10	1080	1135	1195	1250
	50	.04	710	785	865	945
		.06	905	980	1060	1140
		.08	1100	1180	1255	1335
		.10	1295	1375	1450	1530
	60	.04	855	960	1065	1175
		.06	1065	1175	1280	1385
		.08	1280	1385	1495	1600
		.10	1495	1600	1705	1815
70	.04	1005	1140	1280	1415	
	.06	1230	1370	1505	1645	
	.08	1460	1595	1735	1870	
	.10	1685	1825	1960	2100	

\*  $\eta_F = 0.70$

Table A3

Heating Seasonal Operating Cost of Hybrid System with Oil, Gas or Electric Furnace\* in Region V.

Region V	DHR kBtu/h	Cost of electricity \$/kWh	Cost of gas \$/therm	Cost of oil \$/gallon	Add-on heat pump + gas or oil furnace						Add-on heat pump  +  electric furnace
					.26	.39	.52	.65	.78	.91	
					.40	.60	.80	1.00	1.20	1.40	
Region V	30		.04	355	400	440	480	525	565	505	
			.06	490	535	575	620	660	705	760	
			.08	625	670	710	755	795	840	1015	
			.10	765	805	850	890	930	975	1265	
	35		.04	405	460	510	560	610	660	590	
			.06	560	610	660	710	765	815	880	
			.08	715	765	815	865	915	965	1175	
			.10	870	920	970	1020	1070	1120	1470	
	40		.04	460	515	575	635	695	755	670	
			.06	625	685	745	805	865	925	1005	
			.08	795	855	915	975	1035	1095	1345	
			.10	965	1025	1085	1145	1205	1265	1680	
	50		.04	555	635	720	800	885	965	850	
			.06	750	830	915	995	1080	1160	1270	
			.08	940	1025	1105	1190	1270	1355	1695	
			.10	1135	1220	1300	1385	1465	1550	2120	
	60		.04	645	755	865	975	1090	1200	1040	
			.06	860	970	1080	1195	1300	1410	1560	
			.08	1070	1180	1290	1400	1515	1625	2085	
			.10	1285	1385	1505	1615	1725	1835	2605	
70		.04	735	880	1020	1160	1305	1445	1220		
		.06	965	1105	1245	1390	1530	1675	1830		
		.08	1190	1335	1475	1615	1760	1900	2440		
		.10	1420	1560	1700	1845	1985	2125	3050		

\*  $\eta_F = .68, .62, 1$  for oil, gas and electric furnace respectively.

Appendix B

Sample of Heating Seasonal Operating Cost Calculations for a Hybrid System with a Two-Speed Compressor.

Sample calculations of Heating Seasonal Operating Cost for region IV are given below to illustrate the steps involved.

Rating data of add-on heat pump.

		k=1	k=2
$\dot{Q}_{SS}^k$ (62°F)	kBtu/h	35.00	-(1)-
$\dot{Q}_{SS}^k$ (47°F)	kBtu/h	28.50	46.00
$\dot{Q}_{DEF}^k$ (35°F)	kBtu/h	21.00	37.00
$\dot{Q}_{SS}^k$ (17°F)	kBtu/h	14.00	26.00
$\dot{E}^k$ (62°F)	kW	3.00	-(1)-
$\dot{E}^k$ (47°F)	kW	2.70	4.70
$\dot{E}_{DEF}^k$ (35°F)	kW	2.40	4.30
$\dot{E}^k$ (17°F)	kW	1.80	3.90
$C_D^k$		.25	-(2)-

(1) - not required

(2) - depending on control system, may be required. Not required in control case presented here.

### Rating Data for Furnace

The heat pump is assumed to be added to an existing gas, oil or electric furnace. Since no furnace test procedure data is available,  $\eta_F$  are assumed to be 62%, 68% and 100%, respectively, for the gas, oil and electric furnace.

### Control information

The heat pump is the only source of heat down to an outdoor temperature 35°F (1.67°C). Below this temperature it is shut off and furnace is operated. The heat pump will restart after being shutoff [and furnace will cease to operate] when outdoor temperature rises 10°F above the cut-off temperature.

Thus  $T_{OFF} = 35^\circ\text{F}$  (1.67°C) and  $T_{ON} = 45^\circ\text{F}$  (12.4°C)

Also subject to above stipulation the heat pump is designed to cycle between low speed ( $k = 1$ ) to high speed ( $k = 2$ ) when the building load is less than the high speed capacity but greater than the low speed capacity.

### Standard design heating requirements

Use equations (2.1), (2.2) and Table 2.

For region IV

$$\text{DHR}_{\min} = \dot{Q}_{ss}^{k=2}(47^\circ\text{F}) \frac{65-T_{OD}}{60} = 46.00 \frac{65-5}{60} = 46.00 \approx 50.00$$

$$\text{DHR}_{\max} = 2 \dot{Q}_{ss}^{k=2}(47^\circ\text{F}) \frac{65-T_{OD}}{60} = 2 \times 46.00 \frac{65-5}{60} = 92.00 \approx 90.00 \text{ kBtu/h}$$

From Table 2 the standard DHR will then be: 50, 60, 70, 80, 90 kBtu/h.

Calculating Heating Seasonal Operating Cost for DHR = 90 kBtu/h

Table B1 presents sample worksheet for evaluation of HSOC for DHR = 90 kBtu/h in region IV.

The number of heating load hours (HLH) and design outdoor temperature ( $T_{OD}$ ) were obtained from Table 1.

$$HLH = 2250h \quad T_{OD} = 5^{\circ}F$$

Examples of calculations for bin temperature  $T_j = 42^{\circ}F$  are shown below.

column a, b and c: obtain from Table 1.

column d : Since in this case a heat pump is being added to an existing furnace,  $\dot{Q}_F = 1.7 \times DHR = 1.7 \times 90 = 153 \text{ kBtu/h}$ .

column e : the heat pump low speed capacity  $\dot{Q}^{k=1}(T_j)$ , use eq. (2.33)

$$\begin{aligned} \dot{Q}_{(42)}^{k=1} &= \dot{Q}_{SS}^{k=1}(47) + \frac{[\dot{Q}_{SS}^{k=1}(62) - \dot{Q}_{SS}^{k=1}(47)](42-47)}{15} = \\ &= 28.50 + \frac{(35.00 - 28.50)(-5)}{15} = 26.33 \text{ kBtu/h} \end{aligned}$$

column f : the heat pump capacity at the compressor's high speed,  $\dot{Q}^{k=2}(T_j)$ , use eq. (2.34)

$$\begin{aligned} \dot{Q}_{(42)}^{k=2} &= \dot{Q}_{SS}^{k=2}(17) + \frac{[\dot{Q}_{DEF}^{k=2}(35) - \dot{Q}_{SS}^{k=2}(17)][42-17]}{18} \\ &= 26.00 + \frac{[37.00 - 26.00] \times 25}{18} = 41.28 \frac{\text{kBtu}}{\text{h}} \end{aligned}$$

column g : the heat pump power at the compressor's low speed,  $\dot{E}_{SS}^{k=1}(T_j)$ , use eq. (2.35)

$$\begin{aligned}\dot{E}^{k=1}(42) &= \dot{E}_{SS}^{k=1}(47) + \frac{[\dot{E}_{SS}^{k=1}(62) - \dot{E}_{SS}^{k=1}(47)] [42-47]}{15} = \\ &= 2.70 + \frac{[3.00 - 2.70] [-5]}{15} = 2.60 \text{ kW.}\end{aligned}$$

column h : the heat pump high speed power,  $\dot{E}^{k=2}(T_j)$ , use eq. (2.36)

$$\begin{aligned}\dot{E}^{k=2}(42) &= \dot{E}_{SS}^{k=2}(17) + \frac{[\dot{E}_{DEF}^{k=2}(35) - \dot{E}_{SS}^{k=2}(17)] [42-17]}{18} = \\ &= 3.90 + \frac{[4.30 - 3.90] \times 25}{18} = 4.46 \frac{\text{kBtu}}{\text{h}}\end{aligned}$$

column i : the building load,  $BL(T_j)$ , use eq. (2.6)

$$BL(42) = \frac{65-42}{65-5} \times C \times \text{DHR} = \frac{65-42}{65-5} \times .77 \times 90 = 26.57 \frac{\text{kBtu}}{\text{h}}$$

column j : the heating load,  $\frac{n_j}{N} BL(T_j)$ , is evaluated using columns (i) and (c)

$$\frac{n_j}{N} BL(T_j) = .100 \times 26.57 = 2.6565 \text{ kBtu/h}$$

column k :  $\dot{Q}_{SS}^{k=1}(42) < BL(42) < \dot{Q}_{SS}^{k=2}(42)$ , according to control information the heat pump will cycle between low and high speed. This implies case 2.

column l : The compressor cut-off factor  $\delta(T_j)$  use eq. (2.22)

$$\delta'(42) = \frac{1}{2}$$

column m : the heating load factor with the compressor operating on low speed,  $X^{k=1}(T_j)$ , for case 2 use eq. (2.20)

$$X^{k=1}(42) = \frac{\overset{k=2}{\dot{Q}}(42) - BL(42)}{\overset{k=2}{\dot{Q}}(42) - \overset{k=1}{\dot{Q}}(42)} =$$

$$= \frac{41.28 - 26.57}{41.28 - 26.33} = 0.98$$

column n : the heating load factor with the compressor operating on high speed,  $X(T_j)$ , for case 2 use eq. (2.21)

$$X(42) = 1 - X^{k=1}(42) = 1 - 0.98 = 0.02$$

column o,p,q : not appropriate in case 2

column r : the energy used by a heat pump in  $j^{\text{th}}$  bin temperature divided by the total number of bin hours,  $\frac{E_{\text{HP}}(T_j)}{N}$ , use eq. (2.18):

$$\frac{E_{\text{HP}}(42)}{N} = \frac{n_j}{N} \delta'(42) [E^{k=1}(42) X^{k=1}(42) + E^{k=2}(42) X^{k=2}(42)] =$$

$$= .100 \times .5 \times [2.60 \times .98 + 4.46 \times 0.02] = .1314 \text{ kW.}$$

column s : the heating load handled by a furnace in  $j^{\text{th}}$  bin temperature divided by the total number of bin hours,  $\frac{E_F(42)}{N}$ , for case 2 use eq. (2.19)

$$\frac{E_F(42)}{N} = \frac{n_j}{N} BL(42) [1 - \delta(42)] =$$

$$= 2.6570 \times [1 - .5] = 1.3285 \frac{\text{kBtu}}{\text{h}}$$

Once the worksheet calculations are finished heating seasonal operating cost can be evaluated for each combination of electricity and fuel costs for the given region and DHR using equation (2.12).

The following examples are given for electricity cost of \$.05/kWh, cost of oil \$1.00/gallon and cost of gas \$.26/therm:

Hybrid system added to a gas furnace

$$\text{HSOC} = \frac{2250 \times .77 \times 90}{27.6333} \left[ .8504 \times .05 + 19.5792 \times \frac{100}{62} \times \frac{1000}{100,000} \times .26 \right] =$$

$$= \$703 \approx \$705$$

Hybrid System added to an oil furnace

$$\text{HSOC} = \frac{2250 \times .77 \times 90}{27.6333} \left[ .8504 \times .05 + 19.5792 \times \frac{100}{68} \times \frac{1000}{140,000} \times 1.00 \right] = \$1400$$

Hybrid System added to an electric furnace

$$\text{HSOC} = \frac{2250 \times .77 \times 90}{27.6333} \left[ .8504 \times .05 + 19.5792 \times \frac{1000}{3413} \times .05 \right] = \$1858 \approx \$1860$$

The above calculation procedure should be repeated for all standard DHR (given in Table 2) ranging from the minimum to the maximum DHR.

The final results may be also displayed in the format presented in Table 3.

Table B1. Sample Worksheet to Evaluate the Heating Seasonal Energy Input to a Hybrid System with a Two-Speed Compressor or with Two Compressors

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)	(s)				
Temperature bin	Outdoor Temp. $T_j$ (°F)	Fractional Temp. bin hours	$Q_p$ (kBtu/h)	$Q_{k=1}(T_j)$ (kBtu/h)	$Q_{k=2}(T_j)$ (kBtu/h)	$E_{k=1}(T_j)$ (kW)	$E_{k=2}(T_j)$ (kW)	$BL(T_j)$ (kBtu/h)	$\frac{BL}{j}$ (kBtu/h)	CASE	$\phi(T_j)$	$X_{k=1}$	$X_{k=2}$	$k_{k=2}$	$PL_{k=1}$	$PL_{k=2}$	$E_{HP}(T_j)$ (kW)	$E_{HP}(T_j)$ (kBtu/h)				
1	62	.132		35.00	51.25	3.00	5.10	3.47	.4573	1	1	.10	NA	NA	.77	NA	.0506	0.0				
2	57	.111		32.83	49.50	2.90	4.97	9.24	1.0256	1	1	.28	NA	NA	.82	NA	.1104	0.0				
3	52	.103		30.66	47.75	2.80	4.83	15.02	1.5465	1	1	.49	NA	NA	.87	NA	.1618	0.0				
4	47	.093		28.50	46.00	2.70	4.70	20.79	1.9334	1	1	.73	NA	NA	.93	NA	.1964	0.0				
5	42	.100		26.33	41.28	2.60	4.46	26.57	2.6570	2	.5	.98	.02	NA	NA	NA	.1314	1.3285				
6	37	.109	†	21.78	38.22	2.46	4.34	32.34	3.5250	2	.5	.36	.64	NA	NA	NA	.1998	1.7625				
7	32	.126		19.83	35.17	2.30	4.23	38.12	4.8024	4	0	NA	NA	1	NA	1	0.0	4.8024				
8	27	.087	153	17.89	32.11	2.13	4.12	43.89	3.8184	4	0	NA	NA	1	NA	1	0.0	3.8184				
9	22	.055		15.94	29.06	1.97	4.01	49.67	2.7315	4	0	NA	NA	1	NA	1	0.0	2.7315				
10	17	.036	†	14.00	26.00	1.80	3.90	55.44	1.9958	4	0	NA	NA	1	NA	1	0.0	1.9958				
11	12	.026		11.58	22.66	1.65	3.76	61.22	1.5915	4	0	NA	NA	1	NA	1	0.0	1.5915				
12	7	.013		9.17	19.33	1.50	3.63	66.99	0.8708	4	.0	NA	NA	1	NA	1	0.0	0.8708				
13	2	.006		6.75	16.00	1.35	3.50	72.77	0.4365	4	0	NA	NA	1	NA	1	0.0	0.4365				
14	-3	.002		4.33	12.66	1.20	3.37	78.54	0.1570	4	0	NA	NA	1	NA	1	0.0	0.1570				
15	-8	.001		1.92	9.33	1.05	3.23	84.32	0.0843	4	0	NA	NA	1	NA	1	0.0	0.0843				
16	-13	0																				
17	-18	0																				
18	-23	0																				
										27.6333											0.8504	19.5792

REGION IV

$T_{OD} = 5^\circ F$

HLH = 2250

DHR = 90 kBtu/h

$$\sum_j \frac{n_j BL(T_j)}{N} = 27.6333 \frac{kBtu}{h}$$

$$\sum_j \frac{E_{HP}(T_j)}{N} = 0.8504 \text{ kW}$$

$$\sum_j \frac{E_F(T_j)}{N} = 19.5792 \frac{kBtu}{h}$$

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